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An American National Standard

IEEE Guide for the Statistical Analysis of Thermal Life Test Data

Sponsor
Statistics Technical Committee
of the
IEEE Dielectrics and Electrical Insulation Society

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Foreword

(This Foreword is not a part of ANSI/IEEE Std 101-1987, IEEE Guide for the Statistical Analysis of Thermal Life Test Data.)

ANSI/IEEE Std 101-1987 is substantially revised from IEEE Std 101-1972. Extensive revision was deemed necessary to reflect the widespread access of users of this document to advanced calculators and computers. Thus the tedious workbook approach used in previous versions was no longer necessary. However, Annex 3 of this document does contain a detailed worked example to provide guidance to those individuals without advanced calculators or computers.

This document was prepared by the Statistics Technical Committee of the IEEE Dielectrics and Electrical Insulation Society. The members of the working group were as follows:

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T. Claussen
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IEEE Guide for the Statistical Analysis of Thermal Life Test Data

1. Introduction

This revision of IEEE Std 101-1972 describes statistical analyses for data from thermally accelerated aging tests. It explains the basis and use of statistical calculations for an engineer or scientist. Statistical methods discussed in [2] [1] through [6] provide more information. Life test data analysis is dealt with more specifically in [7] through [13]. The subject of this guide is treated extensively in [11] and [12].

1.1 General Background

ANSI/IEEE Std 1-1986 (see Annex 1) discusses the principles for temperature rating of electrical insulation. These principles are carried forward in ANSI/IEEE Std 98-1972 and ANSI/IEEE Std 99-1980 (see Annex 1) which respectively outline test procedures for the experimental estimation of the life of insulating materials and systems. Life test procedures for specific insulating materials and systems are outlined in other IEEE publications (see Annex 1). Insulation life test procedures are also described in ASTM (American Society for Testing and Materials), NEMA (National Electrical Manufacturers Association), and IEC (International Electrotechnical Commission) standards (see Annex 1). Also, proposed life test procedures continue to appear in the literature. All such publications assume that the life of insulation with organic materials is a decreasing function of temperature.

Accelerated test procedures usually call for a number of specimens to be aged at each of several temperatures appreciably above normal operating temperatures. High temperatures are chosen to produce specimen failures (according to specified failure criteria) in typically one week to one year. The test objective is to determine the dependence of median life on temperature from the data and to estimate, by extrapolation, the median life to be expected at service temperature. This guide presents methods for analyzing such data and for comparing test data on different materials.

1.2 The Arrhenius Model

The Arrhenius equation gives the rate of a chemical reaction as a function of temperature. It has been adapted [1] to approximate the relationship between insulation life and temperature as follows.

[1]The numbers in brackets refer to those of the bibliographic entries listed in Section 5..

The Arrhenius equation for a chemical reaction rate is

$$k = D\exp(-E/RT) \tag{1}$$

where

k	= specific reaction rate
E	= activation energy of the reaction (assumed to be constant), in calories/mol, or J/mol or electron volts
R	= Boltzmann gas constant = 1.987 calories/mol/K or 8.314 J/mol/K, or electron volts/K
T	= absolute temperature in degrees Kelvin (273 + temperature in °C)
D	= frequency factor, a quantity that is assumed to be constant; it depends on the number of collisions of the molecules reacting to produce chemical deterioration of the insulation.

The median life L of insulation specimens is assumed to be inversely proportional to the chemical reaction rate k. This yields

$$\log(L) = \text{constant} + ((E/RT)/2.303) \tag{2}$$

where log is the base 10 logarithm throughout.

This “Arrhenius” equation has the algebraic form

$$M(X) = A + BX \tag{3}$$

in which

$M(X)$	= $\log(L)$ = the mean log life
X	= $1/T$
A	= a constant characteristic of the insulation population, test specimen, test method, and failure mode
B	= $E/(2.303R)$, another constant characteristic of the insulation population, test specimen, test method, and failure mode

The coefficients A and B are estimated by fitting the above equation to experimental data. This fitting can be done graphically or, more precisely, by the method of least squares. These methods are presented in sections 2 and 3. Section 3 gives confidence limits that indicate the uncertainty in estimates from data. Throughout, population values are denoted by capital letters (A, B, $M(X)$, etc), and their sample estimates based on experimental data by lower-case letters (a, b, $m(X)$, etc) to distinguish them.

Theoretically, Eq 3 is valid only if a single chemical reaction and failure mode control the insulation failure mechanism. Other reactions may occur, but if they are not dominant, application of the Arrhenius equation may still be valid. The Arrhenius equation application is often valid in practice. Sometimes one reaction and failure mode dominates over a temperature range, but another reaction with a different temperature coefficient and/or failure mode dominates at lower or higher temperatures. Deviations from the simple Arrhenius relation may be caused by different failure modes dominant at different temperatures or by variations in mechanical stress, with temperature, that affect the life. Therefore, while the Arrhenius relation will often fit insulation life-temperature data, it will not always apply. Presented in [8] are valid analyses of such data with two or more failure modes identified by examination of failed specimens.

With these reservations, this guide assumes the Arrhenius equation applies and outlines statistical calculations to fit this equation to test data.

1.3 Test Data Recommendations

Statistical analysis will not compensate for invalid test data. The following recommendations on test procedures ensure correct test data. Some test conditions needed for validity of the Arrhenius relation were mentioned in section 1.2.

Periodic “proof tests” are often used to measure an insulation property that deteriorates. Failure to withstand the specified proof test level is then only detected after a “cycle” of aging. It is more rigorous to treat the failure as if it occurred at the midpoint of the cycle. If there are different modes of failure, the data require special data analysis methods [8].

In planning and carrying out life tests, one should aim toward valid failure time data. One obtains more accurate life estimates at design temperatures from a test with a larger number of specimens at the low end of the test temperature range and fewer at the high end and the middle of this range [11]. It is also best to select the lowest accelerated test temperature as close to the anticipated service use temperature as is possible—consistent with a reasonable estimated test time and enough failures in the allotted test time. Further comments on planning the aging test are given in ANSI/IEEE Std 1-1986 (see Annex 1).

2. Data Analysis

2.1 Graphical Analysis of Data for Individual Temperatures

At any particular test temperature, the log times to failure will be distributed about a mean. A quick, easy, and informative analysis of data at each test temperature is to plot the life times on log-normal probability paper, since the distribution is assumed to be log-normal [11].

First, arrange the n failure times at a temperature in order from lowest to highest (see Table 1). Then calculate the corresponding cumulative probability (cumulative fraction failed), P_j, for the jth failure time:

$$P_j = 100j/(n+1)$$

where j varies from 1 to n. The P_j appear in Table 1.

Then, on log-normal probability paper, plot each failure time versus its P_j, as in the example of Fig 1. Then a line is drawn by eye to best fit the points.

The same procedure is followed for the data at each test temperature. Use log-normal probability paper with enough decades so that the data for all test temperatures fit on the same graph, as shown in Fig 1.

Table 1—Test Data

Temperature (°C)	Specimen No	Life (hours)	Cumulative Probability, P_j
150	1	900	0.09
150	2	900	0.18
150	3	972	0.27
150	4	1260	0.36
150	5	1260	0.45
150	6	1476	0.54
150	7	1476	0.64
150	8	1476	0.73
150	9	1980	0.88
150	10	2196	0.91
175	1	264	0.14
175	2	312	0.29
175	3	312	0.43
175	4	408	0.57
175	5	504	0.71
175	6	552	0.86
200	1	84	0.09
200	2	84	0.18
200	3	108	0.27
200	4	132	0.36
200	5	156	0.45
200	6	156	0.54
200	7	156	0.64
200	8	156	0.73
200	9	204	0.88
200	10	228	0.91

The plotted points for each temperature should be approximately linear. The plots for the different temperatures should be nearly parallel, indicating equal standard deviations of log life. The analysis of such data according to the Arrhenius model is based on the assumptions that the distribution lines for the true populations are straight and parallel. Some departure from strict linearity or parallelism may be acceptable, due to expected randomness in small samples. Gross departure from linearity or parallelism, however, may indicate significant uncontrolled differences in test conditions, specimens, or failure modes at the different temperatures. This would lead to incorrect estimates of life at lower temperatures and incorrect confidence limits.

If the above described plotting is fitted well by straight parallel lines, as illustrated in Fig 1, then the 50% points on the lines give an estimate of the median life at each test temperature. An estimate s of the standard deviation σ at temperature i is the difference between the log of median life and the log of the 16% life on the line for temperature i.

2.2 Calculation of the Mean and Standard Deviation of Data at a Temperature

It is sometimes desirable to analyze the data at individual temperatures to compare two sets of data on different insulations or duplicate tests on the same insulation.

The sample mean and standard deviation of the log life times are calculated for each temperature. Such estimates will generally be close to the graphical estimates. Graphs may, however, reveal important information that the calculations do not. These calculations should be made after converting the life times L_{ij}, of specimen j at temperature i, to their base 10 logs:

$Y_{ij} = \log(L_{ij})$

The log average, $\overline{Y}_i$, of the Y_{ij} values at a temperature i is

$$\overline{Y}_i = \frac{\sum_{j=1}^{n} Y_{ij}}{n_i} \tag{4A}$$

where n_i is the number of specimens at that temperature i, and the sum runs over all n values at the temperature. The subscript j denotes a specimen value within one temperature, i.

It is important to note that if the mean and standard deviation are calculated with the log values (assuming a normal distribution for the log values) that the antilogs of the upper and lower standard deviation interval from the mean will not be symmetric about the mean. The asymmetry will not be great, however, unless the distribution is quite wide.

The sample standard deviation of the log failure times at one temperature is

$$s_i = \sum_{j=1}^{n} (Y_{ij} - \overline{Y}_i)^2 / (n_i - 1)^{0.5} \tag{4B}$$

$$= \left[n_i \sum_{j=1}^{n} Y_{ij}^2 - \left(\sum_{j=1}^{n} Y_{ij} \right)^2 \right] / [n_i(n_i - 1)]^{0.5} \tag{4C}$$

Equation 4B gives explicitly the sum of the squares of the individual deviation Y_{ij} from the mean $\overline{Y}_i$, but Eq 4C is easier to use in calculators and computers, where sums are easily accumulated as data are entered. Some calculators have programs for doing this. Equation 4C is prone to roundoff errors.

The calculated s should be close to the graphical estimate. The calculated values for s from the data of Table 1 are plotted with the failure times in Fig 1. The antilogs of the log mean are plotted at the 50% line, the antilogs of the mean plus the standard deviation at the 84% line, and the antilogs of the mean minus the standard deviation at the 16% line.

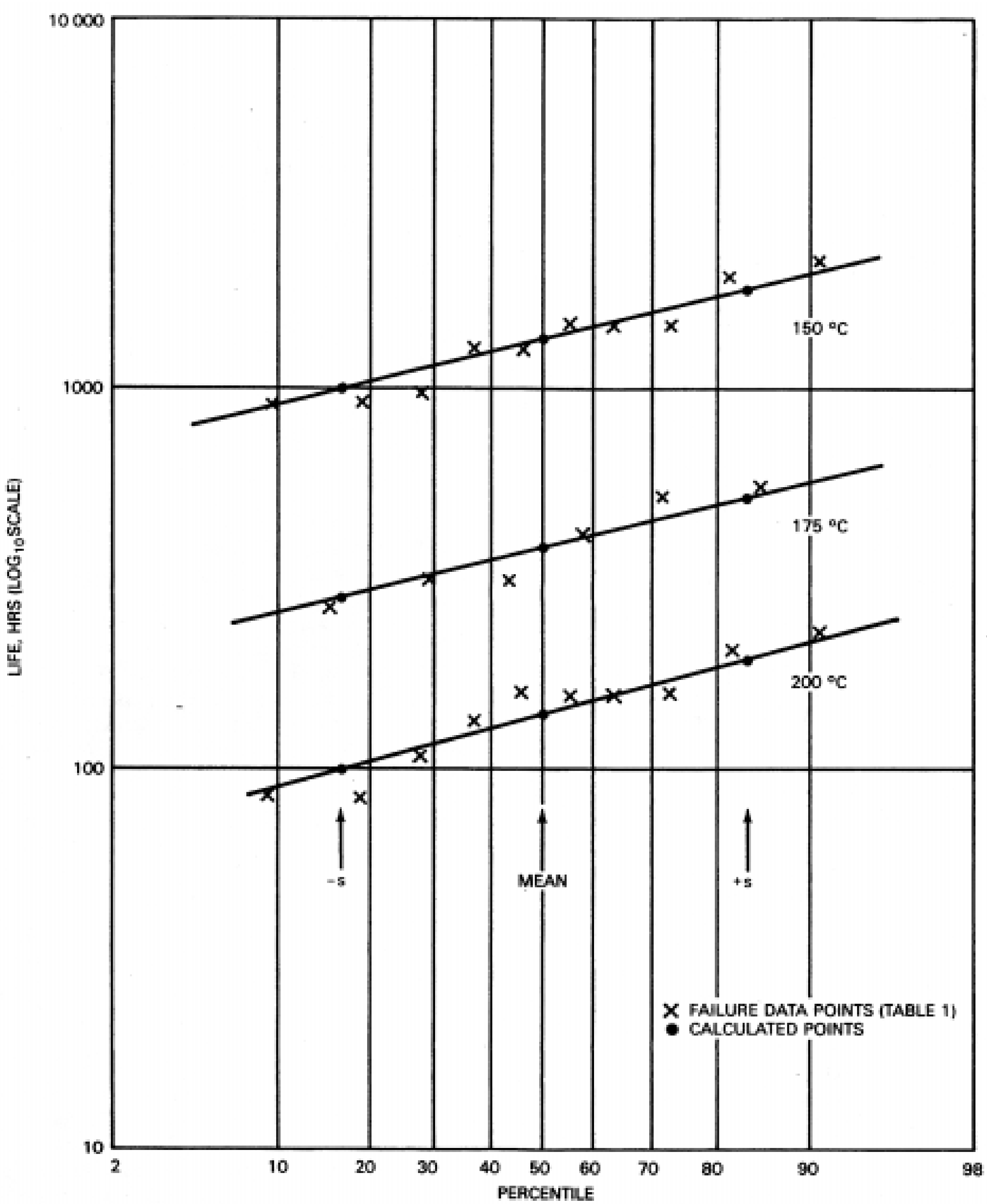

Figure 1—Log Life Versus Normal Probability Graph

Calculation of the standard deviation at individual temperatures permits a statistical comparison of those standard deviations. The statistical significance of their differences can be assessed with Bartlett's test, which is outlined in [11][2].

2.3 Analysis of Incomplete Test Data

It is often desirable to analyze data from a partially completed life test at one or more temperatures [7]. For example, only half of a set of test specimens at one temperature may have failed. Thus, failure times are established for part of the set and a running time for the remainder. Such data with unfailed specimens are called "censored." By analyzing data before all specimens fail, one can save considerable time and cost and still get reliable estimates. Many applications require early analyses. Censored life data at a temperature are called "singly censored" if the unfailed units have a common running time and all the failure times are below that. Data for a temperature are called "multiply censored" if failure and running times differ and are intermixed. The latter type of data result when specimens are put on test at different times.

It is very useful to graph incomplete (censored) test data on log normal probability paper, as in Fig 2. The cumulative probability, P, is calculated for each failure time, as in section 2.1, using the total number of failed and unfailed specimens for n. This yields a plot of the failure times in the lower tail of the sample. A line is drawn through these points parallel to the line(s) for other temperature(s); this line estimates the distribution.

The example in Fig 2 uses the complete data at 200 °C (from Fig 1), the lowest 4 of 6 points at 175 °C, and the lowest 4 of 10 points at 150 °C. The lines were drawn parallel by eye through the data points. The 50% points on these lines give estimates of medians that are close to those obtained in Fig 1 using the complete data.

More extensive graphical and numerical analyses of such incomplete test data are discussed in [7], [9], [10], which in turn give further references.

3. Estimation of the Relationship Between Life and Temperature

3.1 Estimation by Plotting

As the first step, plot the lives (and the antilogs of the averages of log lives, calculated in section 2.2) against temperature on Arrhenius paper, as shown in Fig 3. Arrhenius paper[3] has a log scale for time and a reciprocal Kelvin scale for temperature. Figure 3 was plotted using the data of Table 1.

The plotted points should scatter around a straight line whose algebraic form was given in Eq 3. It is better to fit the line by eye to the median life times obtained from Fig 1 or to the antilogs of the log averages calculated from Eq 4A. This line is used to obtain an estimate of the median life at a temperature, particularly, a design temperature. If a good graphical fit to a straight line is obtained, only slightly greater accuracy in fitting the line is gained by the more objective least squares regression, described in the next section. However, the least squares fit yields much more information, particularly confidence limits.

3.2 Estimation by Numerical Methods

The graphical method does not provide confidence limits or objective statistical comparisons, such as comparing data from two insulation systems.

[2]See Part III, pp 108–111.
[3]This and other special graph papers are available from TEAM, Box 25, Tamworth, NH 03886.

This section discusses, without details, the calculation of the regression line and confidence limits.

The following is assumed:

1) The relation of log life to reciprocal Kelvin temperature is linear over the temperature range of interest (test and design temperatures).
2) The specimen life times are statistically independent (ie, the time to failure of any one specimen does not influence others).
3) Temperature measurement errors are negligible.
4) The specimens are randomly selected from the population of interest.
5) The random variations in log life have a normal distribution with the same standard deviation at all temperatures in the range of interest.

Section 3.3 presents the equations for the calculations. Quantities calculated in early steps are used in the following steps. Ordinarily one would use a standard least squares computer program to do such calculations.

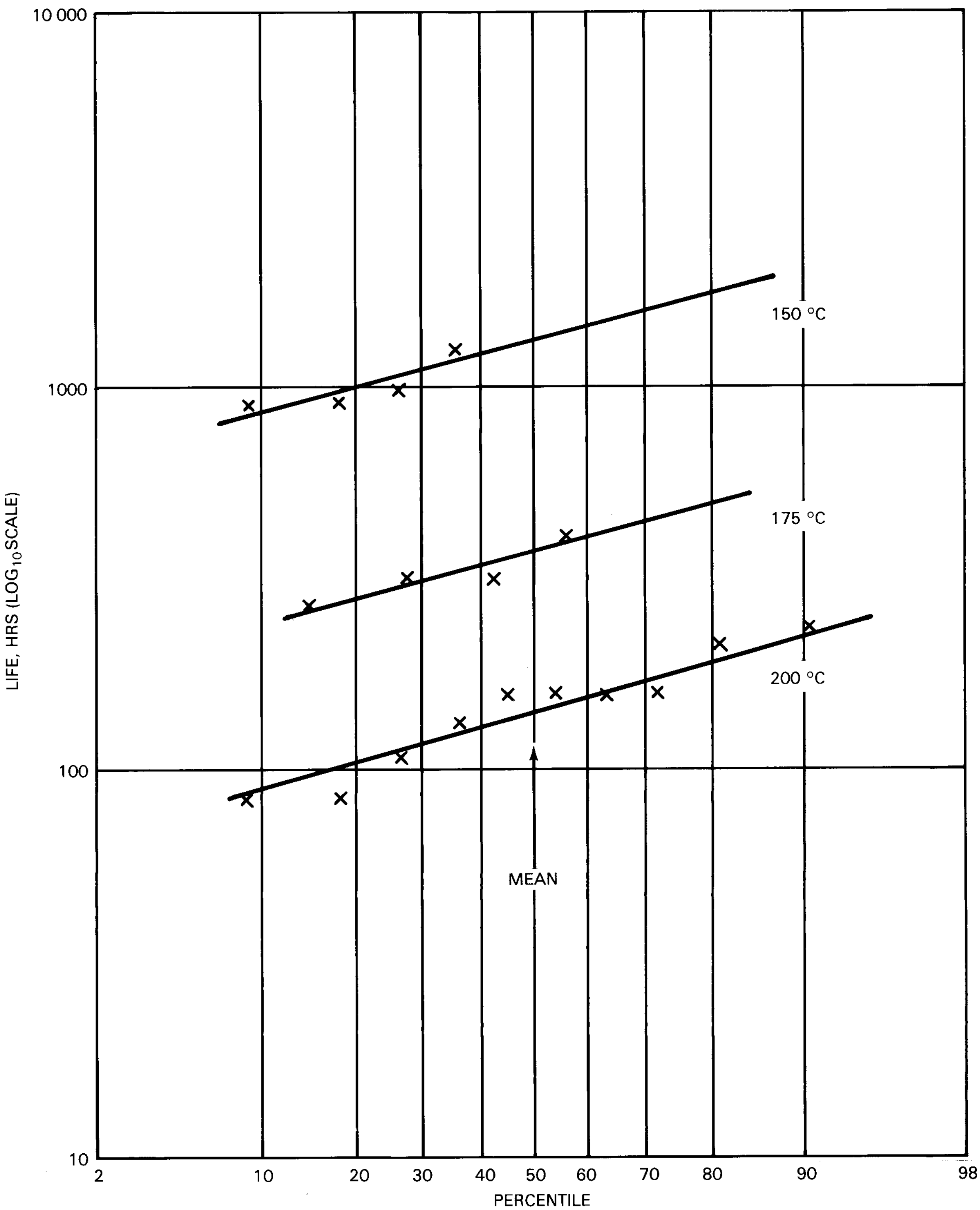

Figure 2—Log Life Versus Normal Probability Graph—Incomplete Data

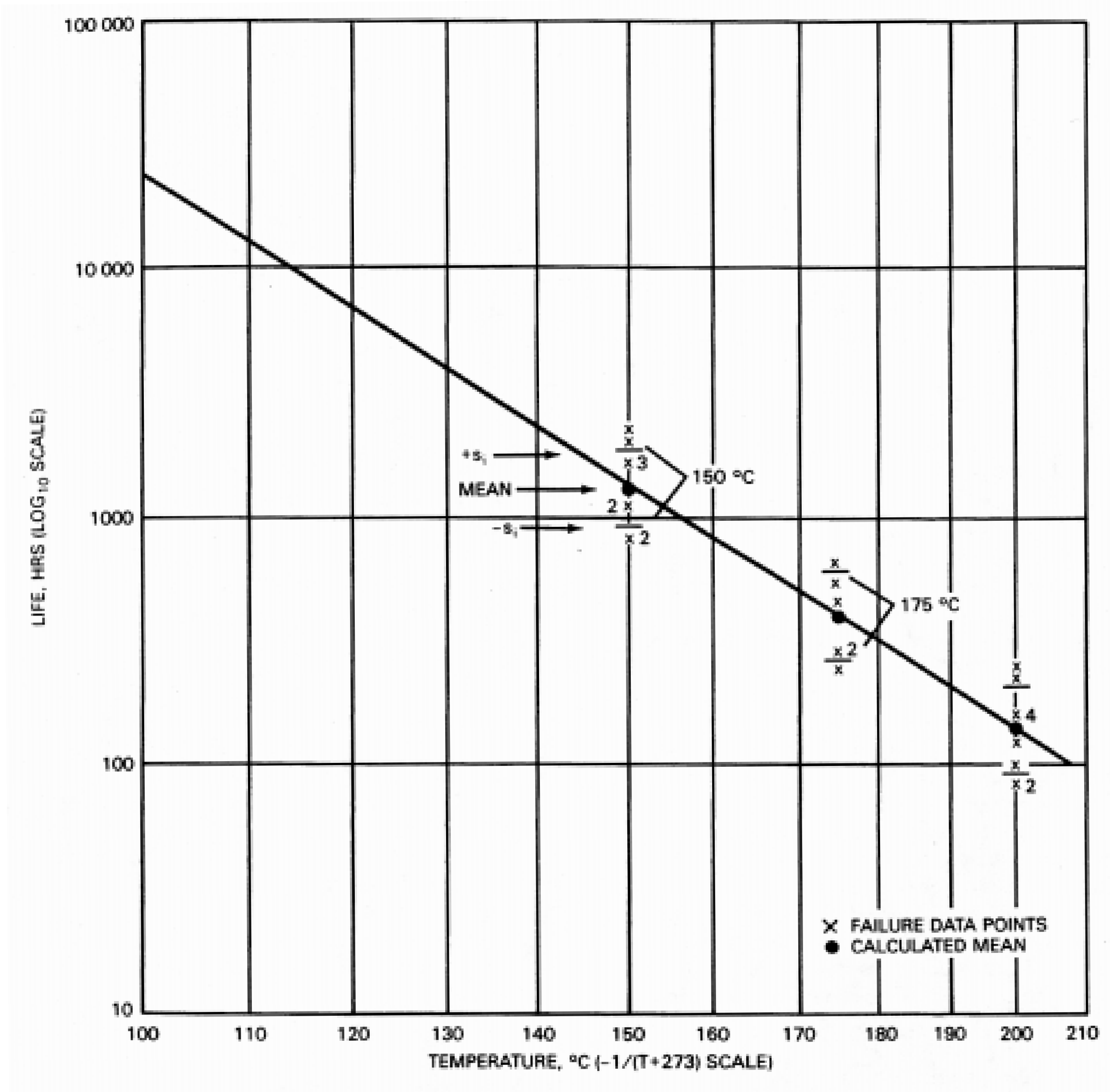

Figure 3—Arrhenius Plot of Life—Temperature Data (from Table 1)

The calculations are illustrated in Annex 2, using the data in Table 1.

The following describes both the calculations and the interpretation of results. Programs for the linear regression calculation are available. Such programs can be adapted by converting the input data to log time and reciprocal Kelvin temperature, respectively. Done by hand, or even with a calculator, the least squares arithmetic is tedious and error prone. Yet one can calculate the quantities by hand following the steps in Annex 2. Also the steps can be programmed into a computer without too much difficulty. It is best to use an existing program, if available, for these calculations [12].

3.2.1 Calculations of the Fitted Line

The least squares fitted line minimizes the sum of the squares of the deviations of the observed log lives from the line on Arrhenius paper.

The calculation of the estimates a and b of the intercept, A, and the slope, B, of Eq 3, is outlined step-wise in sections 3.3.1 through 3.3.4. The calculation is in a form convenient for a programmable calculator or computer or hand calculator. Section 3.3 also includes calculations for the confidence limits for the population line and other population quantities.

There are statistical methods for fitting regression lines to censored data [7], [9], [10]. Graphical methods for this are in section 2.3.

3.2.2 Confidence Limits

The confidence limits indicate the accuracy of estimate of the fitted regression line for mean log life, which is discussed in 3.2.2.1 and in the calculation outline, 3.3. This estimate of the Arrhenius line differs from the population line due to the typically small number of test data points used to calculate it.

The calculation of the confidence intervals uses values from the student's "t" distribution. Several t percentiles are given in Table 2.

Two types of probability limits are described here because of their frequent application and the relative simplicity of their calculation. The first confidence limits applies to the mean life and the second to future individual life values. Which is more appropriate depends on the specific use of the regression line.

3.2.2.1 Confidence Limits for Mean Log Life

Equations for calculating the confidence limits appear in sections 3.3.5 and 3.3.6. The result of a simple calculation appears in Annex 2 and is illustrated in Fig 4. For a specified temperature, these limits will contain the true population mean log life with the selected percent probability (confidence). However, individual life times do fall outside this interval for the mean. The interval indicates the uncertainty between the fitted line and the true Arrhenius line. The percent confidence levels, CL, suggested are 90%, 95%, or 99%. Intervals with other confidence levels may be calculated with the student's t value for the desired percentage from a larger t table. The higher the confidence, the wider the confidence interval.

Confidence limits can be calculated for both sides, or only one side of the population median line. If one is interested in an interval that brackets the population line (above and below the median life), select the factor t for Eqs 8a and 8b from Table 2. However, for electrical insulation, if one is interested in reliability, one may only be interested in a lower confidence limit for population median life. This is calculated by using the student's t' factor (from Table 2) in Eq 8b. Note that the one-sided confidence limit is closer to the median estimate than the two-sided limits—for the same percent confidence. For example, the student's t' values for 97.5% confidence for the one-sided interval equal the t values for 95% confidence for the two-sided interval.

Table 2—Student's "t" Distribution

Values of $t_{N\text{-}2,CL}$ and $t'_{N\text{-}2,CL}$ for CL = 90%, 95%, and 99%					
N-2*	$t'_{N\text{-}2,\,90}$	$t_{N\text{-}2,\,90}$ $t'_{N\text{-}2,\,95}$	$t_{N\text{-}2,95}$	$t'_{N\text{-}2,99}$	$t_{N\text{-}2,99}$
1	3.078	6.314	12.710	31.820	63.660
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771

Values of $t_{N\text{-}2,CL}$ and $t'_{N\text{-}2,CL}$ for CL = 90%, 95%, and 99%

N-2*	$t'_{N\text{-}2,90}$	$t_{N\text{-}2,90}$ $t'_{N\text{-}2,95}$	$t_{N\text{-}2,95}$	$t'_{N\text{-}2,99}$	$t_{N\text{-}2,99}$
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
50	1.298	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
80	1.292	1.664	1.990	2.374	2.639
100	1.290	1.660	1.984	2.365	2.626
200	1.286	1.653	1.972	2.345	2.601
500	1.283	1.648	1.965	2.334	2.586
∞	1.282	1.645	1.960	2.326	2.576

*Referred to as the “degrees of freedom”

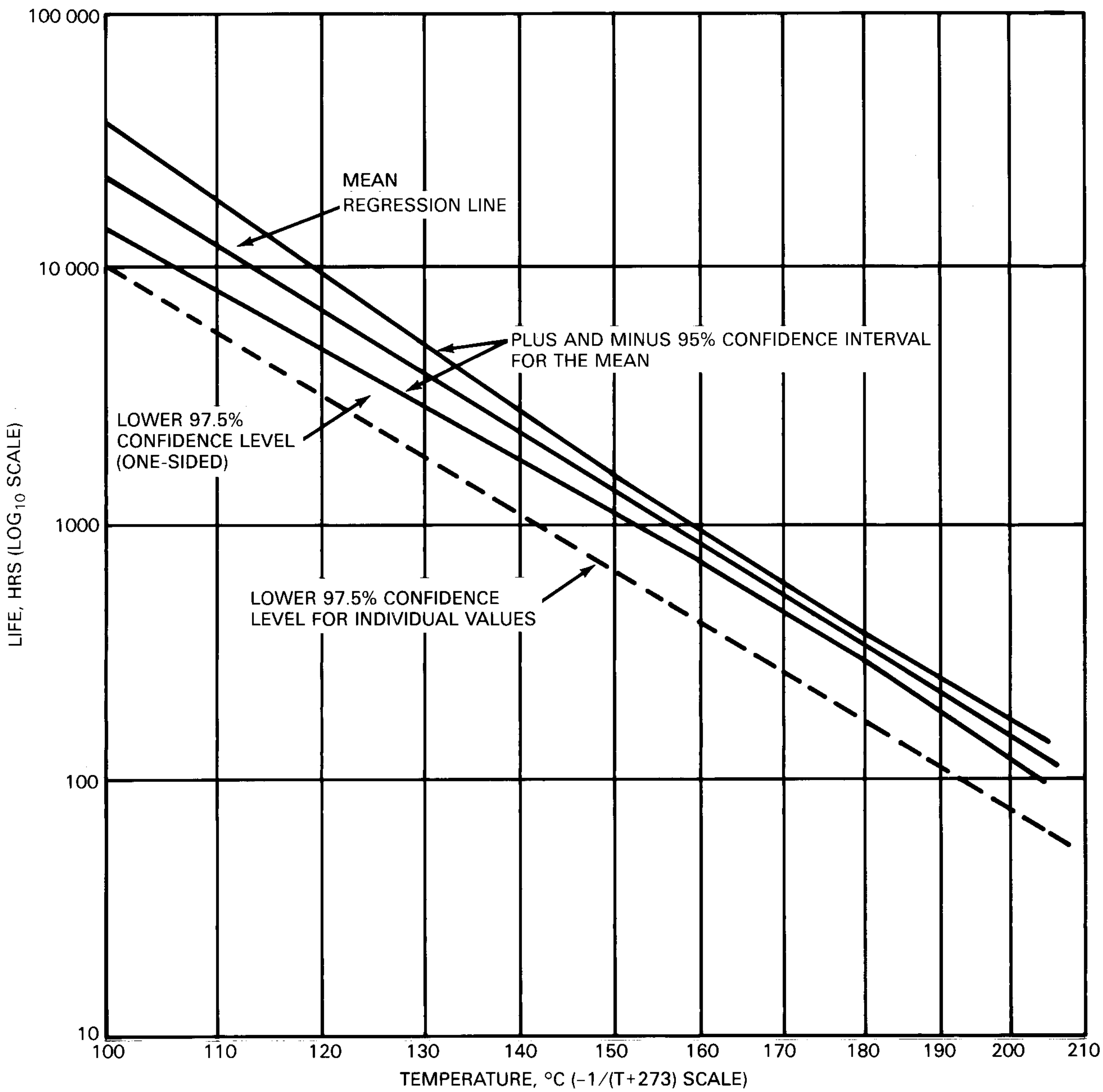

Figure 4—Graph of Arrhenius Regression Line and Confidence Limits (Table 1 Data)

It is, of course, customary and convenient to convert the limits for mean log life back to their corresponding times in hours or other original time units.

3.2.2.2 Prediction Limits for the Log Life of Individual Future Units

The second type of probability limits enclose the log life of a single future specimen at a specified temperature, T_c. As might be expected, these limits are significantly wider than those for the mean log life. Equations for these limits appear in section 3.3.6.

It is probably prudent for persons using these statistical calculations to consider these confidence limits, since they indicate the breadth of variation that can be expected for an individual specimen life from the mean. Since the lower limit is usually more important, the student's t' value might preferably be selected for this calculation, using Eq 9B. If desired, the two-sided limits can also be considered and calculated using the student's t value in Eqs 9A and 9B.

Unlike the confidence interval for the mean (see Eqs 8a and 8b), the prediction interval for an individual future life does not become arbitrarily small as the number of test points is made increasingly large.

3.3 Calculation Outline

3.3.1 Data Conversion to Log Time and Reciprocal Kelvin Temperature

1) Convert each specimen's Celsius test temperature T to reciprocal Kelvin temperature:

$X = 1/(T + 273)$

2) Convert each specimen's failure life time L to $Y = \log(L)$ (base 10).

These conversions can be done with a calculator or computer as the data are entered.

3.3.2 Accumulation of Sums

For the specimen X, Y values, accumulate the following sums:

$\Sigma X, \Sigma Y, \Sigma X^2, \Sigma Y^2, \Sigma(XY)$

where these sums are calculated over the total number of specimens, N, from all test temperatures. The number of specimens at each temperature need not be the same.

Store these for future calculations. They must be accurate to at least seven significant figures to avoid roundoff error in the subsequent calculations. Some calculators calculate these sums as the (X, Y) pairs are entered.

3.3.3 Estimates of the Arrhenius Coefficients

The sample estimate of the population slope B is

$$b = \frac{N\Sigma(XY) - (\Sigma X)(\Sigma Y)}{N\Sigma(X^2) - (\Sigma X)^2} \quad (5A)$$

The sample estimate of the population intercept A is

$$a = (\Sigma Y - b\Sigma X)/N \quad (5B)$$

Store these values for future use.

3.3.4 Calculation of the Fitted Regression Line

For a selected temperature T_c, the sample estimate of the population mean log life, $M(T_c)$, is calculated from

$$m(T_c) = a + b[1/(T_c + 273)] \quad (5C)$$

The antilog of $m(T_c)$ is the estimate of median life in hours at T_c. Although only two mean estimates (preferably far apart) are algebraically necessary to define the line on an Arrhenius plot, it is usually recommended or required that tests be run to establish the mean at three or four temperatures, since this permits checking the linearity of the fit of the data to the Arrhenius relation.

3.3.5 Calculation of Confidence Limits

Calculate the estimate of the standard deviation σ of the log life:

$$s = \{\Sigma[Y-(a+bX)]^2/(N-2)\}\}^{0.5} \quad (6)$$

The sum is taken over all N specimens.

For a selected temperature, T_c, where $X_c = 1/(T_c + 273)$, calculate the factor:

$$V(T_c) = \frac{\{X_c-(\Sigma X/N)\}^2}{\Sigma X^2-(\Sigma X)^2/N} \quad (7)$$

The sums were calculated in section 3.3.2. σ is the same for all temperatures, but $V(T_c)$ must be calculated for each temperature, T_c, for which confidence limits are desired. Store these factors for future use.

The two-sided upper, $m_U(T_c)$, and lower $m_L(T_c)$ confidence limits for the mean log life $M(T_c)$ are

$$m_U(T_c) = (a+bX_c)+t_{N-2}s\{(1/N)+V(T_c)\}^{0.5} \quad (8A)$$

$$m_L(T_c) = (a+bX_c)+t_{N-2}s\{(1/N)+V(T_c)\}^{0.5} \quad (8B)$$

where t_{N-2} is the value of the t statistic for the selected confidence level (for example: 90% or 95%) from Table 2.

For just a lower (one-sided) confidence limit, use factor t' and Eq 8b. Refer to the explanation in section 3.2.2.1.

The confidence limits would normally be calculated for selected service and test temperatures, since the limits are not linear functions of X. (See Fig 4 for a plot of such limits and the calculation example in Annex 2.)

It is usually desirable to convert these log life limits back to their corresponding life times, by taking their antilogs.

3.3.6 Calculation of Predicted Limits for a Single Future Failure Time Y.

These limits are wider than those for the regression line for mean life. See the discussion of this quantity in section 3.2.2.2.

Calculate:

$$Y_U(T_c) = (a+bX_c)+t_{N-2}s\{1+(1/N)+V(T_c)\}^{0.5} \quad (9A)$$

$$Y_L(T_c) = (a+bX_c)-t_{N-2}s\{1+(1/N)+V(T_c)\}^{0.5} \quad (9B)$$

where t_{N-2} is the same as in section 3.3.5. If only the lower prediction limit, $Y_L(T_c)$ for an individual future value is desired, t' is used in the equation.

It is usually desirable to convert these values of the log life limits back to life times, by taking their antilogs.

The prediction limits for an individual future value of log life, analogous to the confidence limits for mean log life, may be plotted on the same graph with the fitted regression line. The lower prediction curve is illustrated in Fig 4 for the data from Table 1.

3.3.7 Linearity

The statistical calculations which fit Eq 2 to the life versus temperature data assume that the straight line (Arrhenius Equation) correctly fits such data. Changes in the slope of the line for log life versus reciprocal absolute temperature may indicate changes in aging mechanisms. Since extrapolation based on such data without additional considerations may be of questionable validity, it is necessary to determine that acceptable linearity exists. See Annex 3 for a discussion on linearity test procedures.

4. Comparison of Two Sets of Data

4.1 Comparison of Two Means at a Temperature

The following method compares two sample means (averages) of log life at the same temperature and calculated as in section 2.2. A "t" test is used to assess whether the higher log (mean) is statistically significantly higher than the lower log (mean).

$m_1 (T_c)$ = higher mean log life at temperature T_c
$m_2(T_c)$ = lower mean log life at temperature T_c
s_1 = standard deviation of the data with the higher mean
s_2 = standard deviation of the data with the lower mean
n_1 and n_2= representative number of data points used to calculate each mean

These quantities are calculated from Eqs 4A, 4B, and 4C. The comparison assumes that the two population standard deviations are not too different. With a difference of less than about 10%, it is reasonable to consider the two standard deviations the same.

From the two standard deviations a "pooled" standard deviation is calculated with the following equation:

$$s_p = \left[\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}\right]^{0.5} \qquad (10)$$

The t statistic for the difference between $\bar{Y}_1$ and $\bar{Y}_2$ is

$$t = \frac{m_1(T_c) - m_2(T_c)}{S_p\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}^{0.5}} \qquad (11)$$

The number of degrees of freedom for this t statistic is $n_1 + n_2 - 2$.

To find the significance level of this statistic, enter the t table (Table 2) at the row for the above degrees of freedom. Go across to a value of t that most closely matches that calculated above for the difference. One-hundred percent minus the percentage heading on this column for t' gives the significance level of the statistic. If this significance level is small (<5%), then there is convincing evidence of a difference in the means.

Preferably, a confidence interval for the difference between the two population means may be calculated for a specified confidence level (for example, 95%) using the corresponding t value:

$$[m_1(T_c) - m_2\ (T_c)] \pm t\ s_p \left(\frac{1}{n_1} + \frac{1}{n_2}\right)^{0.5} \tag{12}$$

If this interval encloses zero, the two sample means do not differ significantly. If the interval does not enclose zero, then the sample means differ significantly; that is, they differ convincingly. Thus one acts as if the mean log lives for the two insulations are different. $m_1\ (T_c) - m_2\ (T_c)$ is an estimate of how much they differ.

4.2 Comparison of Two Arrhenius Lines at a Temperature

Thermal life tests are often used to compare two different systems or materials. Arrhenius lines are fitted to data for both materials. There is a question whether one insulation system is significantly better than the other, ie, are the two lines significantly different. The observed difference might be due to chance variation. To assess whether or not there is a convincing difference, use the following comparison between the two sets of data. The calculation is similar to that for comparing two averages.

The comparison between the fitted lines is made at one or more temperatures, usually at a service temperature. It may also be useful to compare at a temperature in the test range.

The calculation utilizes some quantities obtained in section 3.3.5, namely the standard deviations s_1 and s_2 (Eq 6) and $V_1\ (T_c)$ and $V_2(T_c)$ (Eq 7). These are subscripted 1 and 2 to identify the insulation. The s values are independent of temperature, but the $V(T_c)$ values vary with temperature and must be calculated for each selected temperature, T_c, where the comparison is made. (Annex 2 illustrates the variation in $V(T_c)$ for five temperatures.)

First calculate a pooled standard deviation s_p for the two insulations:

$$s_p = \left[\frac{(N_1 - 2)s_1^2 + (N_2 - 2)s_2^2}{(N_1 + N_2 - 4)}\right]^{0.5} \tag{13}$$

where N_1 and N_2 are the numbers of specimens used to estimate the respective fitted lines. Then calculate the mean log life $M(T_c)$ for each line at the selected temperature, T_c. These estimates are calculated using Eq 5c, using their respective coefficients, a and b

Finally, calculate the statistic:

$$t = \frac{m_1(T_c) - m_2(T_c)}{s_p\left[\frac{1}{N_1} + V_1(T_c) + \frac{1}{N_2}V_2(T_c)\right]^{0.5}} \tag{14}$$

Enter a table of t percentiles (Table 2) at the row for $N1 + N2$-4 degrees of freedom. Go across this row to the t percentile most nearly equal to the t statistic. One hundred percent minus the percentage at the head of this column gives the "significance level" of the t statistic.

Alternatively, confidence limits for the difference between the log means at the selected temperature (T_c) may be calculated for a selected confidence level and corresponding t value:

$$n_1(T_c) - m_2(T_c) \pm t s_p\left[\frac{1}{N_1} + V_1(T_c) + \frac{1}{N_2} + V_2(T_c)\right]^{0.5} \tag{15}$$

If these limits enclose zero, the two log means do not differ significantly. If the limits do not enclose zero, the means are significantly different.

5. Bibliography

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[4]Available from Distribution Unit, GE Research and Development Center, PO Box 43, Bldg 5, Schenectady, NY 12305.
[5] See footnote 4.

Annex 1

(Informative)

List of Thermal Life Test and Related Standards for Electrical Insulation

(These Appendixes are not part of ANSI/IEEE Std 101-1987, IEEE Guide for the Statistical Analysis of Thermal Life Test Data, but are included for information only.)

The following are lists of IEEE and IEC standards or guides to which apply the statistical techniques described in this guide.

A1.1 IEEE Standards

[A1] ANSI/IEEE Std 1-1986, IEEE Standard General Principles for Temperature Limits in the Rating of Electrical Equipment and for the Evaluation of Electrical Insulation.[6]

[A2] ANSI/IEEE Std 98-1984, IEEE Standard for the Preparation of Test Procedures for the Thermal Evaluation of Solid Electrical Insulating Materials.

[A3] ANSI/IEEE Std 99-1980, IEEE Recommended Practice for the Preparation of Test Procedures for the Thermal Evaluation of Insulation Systems for Electrical Equipment.

[A4] ANSI/IEEE Std 117-1974 (R1985), IEEE Standard Test Procedure for the Evaluation of Systems of Insulating Materials for Random-Wound AC Electric Machinery.

[A5] ANSI/IEEE Std 275-1981, IEEE Recommended Practice for Thermal Evaluation of Insulation Systems for AC Electric Machinery Employing Form-Wound Pre-Insulated Coils, Machines Rated 6900 V and Below.

[A6] ANSI/IEEE Std 304-1977 (R1982), IEEE Standard Test Procedures for Evaluation and Classification of Insulation Systems for DC Machines.

[A7] ANSI/IEEE Std 429-1972, IEEE Standard Test Procedure for the Evaluation of Sealed Insulation Systems for AC Electric Machinery Employing Form-Wound Stator Coils.

[A8] ANSI/IEEE Std 434-1973 (R1985), IEEE Guide for Functional Evaluation of Insulation Systems for Large High-Voltage Machines.

[A9] ANSI/IEEE Std 620-1987, IEEE Guide for Construction and Interpretation of Thermal Limit Curves for Squirrel-Cage Motors Over 500 hp.

[A10] ANSI/IEEE Std 930-1987, IEEE Guide for the Statistical Analysis of Voltage Endurance Data for Electrical Insulation.

[A11] ANSI/IEEE C57.91-1981, IEEE Guide for Loading Mineral-Oil-Immersed Overhead and Pad-Mounted Distribution Transformers Rated 500 kVA and Less with 65 °C or 55 °C Average Winding Rise.

[A12] ANSI/IEEE C57.92-1981, IEEE Guide for Loading Mineral-Oil-Immersed Power Transformers up to and Including 100 MVA with 55 °C or 65 °C Winding Rise.

[6] ANSI/IEEE publications can be obtained from the Sales Department, American National Standards Institute, 1430 Broadway, New York, NY 10018, or from the Institute of Electrical and Electronics Engineers, Service Center, 445 Hoes Lane, Piscataway, NJ 08854-4150.

[A13] IEEE Std 266-1969, (R1981), IEEE Test Procedures for Evaluation of Insulation Systems for Electronics Power Transformers.

A1.2 IEC Standards

[A1] IEC 172 (1987), Test Procedure for the Determination of the Temperature Index of Enamelled Winding Wires.[7]

[A2] IEC 216, Guide for the Determination of Thermal Endurance Properties of Electrical Insulating Materials.

[A3] IEC 216-1 (1987), Part 1: General Guidelines for Aging and Evaluation of Test Results.

[A4] IEC 216-2 (1984), Part 2: List of Materials and Available Tests.

[A5] IEC 216-3 (1980), Part 3: Statistical Methods.

[A6] IEC 216-4 (1980), Part 4: Instructions for Calculating the Thermal Endurance Profile.

[A7] IEC 290 (1969), Evaluation of the Thermal Endurance of Electrical Insulating Varnishes by the Helical Coil Bond Test.

[A8] IEC 370 (1971), Test Procedure for Thermal Endurance of Insulating Varnishes--Electrical Strength Method.

[7]IEC publications can be obtained from Bureau Central de la Commission Electrotechnique Internationale, 3 rue de Varemb???, 1211 Geneve 20, Suisse (Switzerland). They may also be obtained from the American National Standards Institute, 1430 Broadway, New York NY 10018.

Annex 2

(Informative)

Example Calculations

This appendix presents an example of the calculations of statistical quantities discussed above. The data from Table 1, discussed in section 2.2, are used here to calculate the fitted line and its confidence limits.

The calculations employ Table A2-1. This table organizes conversions of the input temperature and failure times to *X* and *Y* values for calculations. The table is convenient for a hand calculator. Fill in the columns in the table. Start with the conversion to *X* and *Y* in columns 5 and 6. Then calculate the squares and products in columns 7, 8, and 9. The sums of these columns are then inserted into the equations for the desired quantities.

Computer calculations are usually similarly organized, but the intermediate values are usually not printed out unless they might be useful for further calculations. The program listed in [12][8] is, however, specifically designed for this application.

If only the fitted line is calculated, the sums for individual temperatures, as shown in Table A2-1, are not needed. These sums may have already been calculated in analyzing data at individual temperatures as in Table 1. Also, some are used in further statistical calculations. So it is suggested that they be calculated.

[8]See bibliography, Section 5..

Table A2-1 —Data for Example Calculation

I	*J*	*TC*	Hrs	*X = 1*/(273+*T*)	*Y* = log(*L*)	*X* Squared	*Y* Squared	*X* Times *Y*
1	1	150	900	2.36406619E-03	2.95424251	5.58880897E-06	8.72754882	6.98402485E-03
1	2	150	900	2.36406619E-03	2.95424251	5.58880897E-06	8.72754882	6.98402485E-03
1	3	150	972	2.36406619E-03	2.98766626	5.58880897E-06	8.92614972	7.06304082E-03
1	4	150	126 0	2.36406619E-03	3.10037055	5.58880897E-06	9.61229752	7.32948119E-03
1	5	150	126 0	2.36406619E-03	3.10037055	5.58880897E-06	9.61229752	7.32948119E-03
1	6	150	147 6	2.36406619E-03	3.16908636	5.58880897E-06	10.0431083	7.49192993E-03
1	7	150	147 6	2.36406619E-03	3.16908636	5.58880897E-06	10.0431083	7.49192993E-03
1	8	150	147 6	2.36406619E-03	3.16908636	5.58880897E-06	10.0431083	7.49192993E-03
1	9	150	198 0	2.36406619E-03	3.29666519	5.58880897E-06	10.8680014	7.79353473E-03
1	10	150	219 6	2.36406619E-03	3.34163234	5.58880897E-06	11.1665067	7.89984004E-03
Sums for *T*1 —			—	0.0236406619	31.242449	5.58880897E-05	97.7696755	0.0738592174
2	1	175	264	2.23214286E-03	2.42160393	4.98246173E-06	5.86416558	5.40536591E-03
2	2	175	312	2.23214286E-03	2.49415459	4.98246173E-06	6.22080715	5.56730936F-03
2	3	175	312	2.23214286E-03	2.49415459	4.98246173E-06	6.2280715	5.56730936E-03
2	4	175	408	2.23214286E-03	2.61066016	4.98246173E-06	6.8155465	5.82736643E-03
2	5	175	504	2.23214286E-03	2.70243054	4.98246173E-06	7.30313081	6.03221102E-03
2	6	175	552	2.23214286E-03	2.74193908	4.98246173E-06	7.51822992	6.12039973E-03
Sums for *T*2 —			—	0.0133928571	15.4649429	2.98947704E-05	39.9426871	0.0345199618
3	1	200	84	2.11416491E-03	1.92427929	4.46969324E-06	3.70285078	4.06824373E-03
3	2	200	84	2.11416491E-03	1.92427929	4.46969324E-06	3.70285078	4.06824373E-03
3	3	200	108	2.11416491E-03	2.03342375	4.46969324E-06	4.13481217	4.29899314E-03
3	4	200	132	2.11416491E-03	2.12057393	4.46969324E-06	4.4968338	4.48324298E-03
3	5	200	156	2.11416491E-03	2.1931246	4.46969324E-06	4.80979551	4.63662706E-03
3	6	200	156	2.11416491E-03	2.1931246	4.46969324E-06	4.80979551	4.63662706E-03
3	7	200	156	2.11416491E-03	2.1931246	4.46969324E-06	4.80979551	4.63662706E-03
3	8	200	156	2.11416491E-03	2.1931246	4.46969324E-06	4.80979551	4.63662706E-03
3	9	200	204	2.11416491E-03	2.30963017	4.46969324E-06	5.33439151	4.88293904E-03
3	10	200	228	2.11416491E-03	2.35793485	4.46969324E-06	5.55985675	4.9850631E-03
Sums of *T*3 —			—	0.021141649	21.4426197	4.46969324E-05	46.1707778	0.45333234
Sum for all *T* —			—	0.058175681	68.1500115	1.30479793E-04	183.88314	0.153712413

A2.1 Calculation of the Coefficients of the Linear Regression

The slope estimate is[9]

$$b = \frac{N\Sigma(XY) - (\Sigma X)(\Sigma Y)}{N\Sigma(X^2) - (\Sigma X)^2} \qquad (5a)$$

All of these sums are taken over all the *N* specimens (26 in this case) and appear in Table A2-1 at the bottom. Inserting the sums into the equation gives

$$b = \frac{26\ (0.153712413) - (0.0581751681)\ (68.1500115)}{26\ (0.000130479793) - (0.0581751681)^2}$$

$$= \frac{3.996522738 - 3.964638375}{0.0033924746 - 0.0033843502} = 3924.5$$

Note that the differences in the numerator and denominator are in the third and fourth significant figure. Thus it is essential to work with at least seven significant figures in intermediate calculations.

The estimate of the intercept of the Arrhenius line is[10]

$$a = [\Sigma Y - b(\Sigma X)]/N \qquad (5b)$$

Inserting b and ΣX and ΣY gives

$$\begin{aligned} a &= [68.1500115 - (3924.5)\ 0.0581751681]/26 \\ &= -6.15994 \end{aligned}$$

A2.2 Calculation of Points on the Fitted Line

For a selected temperature, the value of the fitted point is[11]

$$m(T_c) = a + bX_c \qquad (5c)$$

By inserting values of $X_c = 1/(273 + T_c)$ for selected temperatures, T_c, the corresponding estimates of mean log life are calculated. Take the antilog to get the median life estimate in hours. For example, for 150 °C,

$$\begin{aligned} m(150) &= -6.15994 + (3924.5)[1/273 + 150] \\ &= 3.11784 \end{aligned}$$

Its antilog is the estimate of median life and is 1312 hours.

[9]This equation is taken from section 3.3.3 on p 16.
[10]See footnote 9.
[11]See footnote 9.

Points on the fitted line are given below.

T_c	$m(T_c)$	antilog $m(T_c)$
100 °C	4.36151	22 988 hrs
130	3.57821	3379
150	3.11784	1311
175	2.60010	398
200	2.13710	137

Although only two points are needed to determine the line, values of $m(T_c)$ at the other temperatures are useful.

A2.3 Calculation of the Standard Deviation About the Fitted Line

The standard deviation s of the data points about the fitted line is calculated with[12]

$$s = \{\Sigma[Y - a - bX]^2 / (N - 2)\}^{0.5} \tag{6}$$

The sums are taken over all test points at all temperatures. This calculation is shown in Table A2-2. The temperatures are tabulated in column 1, the measured Y values in column 2, and the calculated m_i values are in column 3. In column 4 are the calculated values of differences between columns 2 and 3. Finally, in column 5 are the squares of the differences in column 4. Insertion of the sum of the fifth column into Eq 6 yields the value

$$\begin{aligned} s &= [0.438733372/(26\text{-}2)]^{0.5} \\ &= 0.135206 \end{aligned}$$

A2.4 Calculation of V(T_c)

The quanitity $V(T_c)$ must be calculated for each temperature for which confidence limits are desired:[13]

$$V(T_c) = [X_c - (\Sigma X)/N]^2 / [\Sigma X^2 - (\Sigma X)^2 / N] \tag{7}$$

An example calculation for 150 °C is given here. Use values of the appropriate sums from Table A2-1 to calculate

$$V(T_c) = \frac{[(0.0023641) - (0.0581752/26)]^2}{(0.000130479) - ((0.0581752)^2/26)}$$

$$= \frac{(0.0023641 - 0.00223751)^2}{(0.000130479 - 0.000130167)} = 0.0513$$

Note that here the difference in the denominator is in the 4th and 5th significant figure or the 7th and 8th figure after the decimal point. This emphasizes the need to carry 8 or 9 significant figures in the sums of X and Y, etc.

The following is a short table of $V(T_c)$ values:

[12]This equation is taken from section 3.3.5 on p 16.
[13]See footnote 12.

T_c	$V(T_c)$
100	0.6273
130	0.1903
150	0.0513
175	0
200	0.0486

A2.5 Calculation of Confidence Limits for Mean Log Life

Now all of the appropriate quantities are available for the calculation of the confidence limit values in log form. The appropriate *t* and *t*' values for 95% confidence are obtained for *N*-2 (= 24 here) degrees of freedom from the *t* table in Table 2. The upper and lower (two-sided) confidence limits for the regression line are[14]

$$m_U(T_c) = (a + bX_c) + t_{N-2}s((1/N) + V(T_c))^{0.5} \tag{8a}$$

$$m_L(T_c) = (a + bX_c) - t_{N-2}s((1/N) + V(T_c))^{0.5} \tag{8b}$$

For 150 °C, necessary values have already been calculated in this appendix. Insertion of these values yields

$$\begin{aligned} m_U(150) &= (-6.15994 + (3924.5)/(273.16 + 150)) \\ &\quad + 2.064(0.135206)[(1/26) + 0.0513]^{0.5} \\ &= 3.11784 + 0.08361 = 3.20145 \end{aligned}$$

and Eq 8b yields

$$m_L(150) = 3.11784 - 0.08361 = 3.03423$$

The antilogs of $m_U(150)$ and $m_L(150)$ are then taken to get the limit in hours. The corresponding confidence limits for median life at 150 °C are

$L_{0.95U}$ = 1590 hours

$L_{0.95L}$ = 1082 hours

For a comparison, the estimate of median life calculated above is 1311 hours. Such confidence limits have been calculated for other temperatures for the data of Table 1. They are plotted in Fig 4. Note that the lower 95% (two-sided) confidence limit calculated above is the one-sided 97.5% confidence limit.

[14]See footnote 12 on p 24.

Table A2-2 —Data for Confidence Interval Calculation

Temperature	$Y_{ij} = \log L_{ij}$	$m_i = a + bX$	$Y_{ij} - mi$	$(Y_{ij} - mi)^2$
150 °C	2.95424251	3.11783922	-0.163596708	0.026763883
150 °C	2.95424251	3.11783922	-0.163596708	0.026763883
150 °C	2.98766626	3.11783922	-0.130172953	0.0169449977
150 °C	3.10037055	3.11783922	-0.174686732	3.05154543E-04
150 °C	3.10037055	3.11783922	-0.174686732	3.05154543E-04
150 °C	3.16908636	3.11783922	0.0512471395	2.6262693E-03
150 °C	3.16908636	3.11783922	0.0512471395	2.6262693E-03
150 °C	3.29666519	3.11783922	0.178825972	0.0319787281
150 °C	3.34163234	3.11783922	0.223793118	0.0500833598
175 °C	2.42160393	2.60010475	-0.17850082	0.0318625427
175 °C	2.49415459	2.60010475	-0.105950153	0.0112254348
175 °C	2.49415459	2.60010475	-0.105950153	0.0112254348
175 °C	2.61066016	2.60010475	0.0105554163	1.11416814E-04
175 °C	2.70243054	2.60010475	0.10232579	0.0104705672
175 °C	2.74193908	2.60010475	0.141834332	0.0201169776
200 °C	1.92427929	2.13709908	-0.212819794	0.0452922646
200 °C	1.92427929	2.13709908	-0.212819794	0.0452922646
200 °C	2.03342375	2.13709908	-0.103675324	0.0107485729
200 °C	2.12057393	2.13709908	-0.0165251493	2.73080561E-04
200 °C	2.12057393	2.13709908	0.0560255181	3.13885868E-03
200 °C	2.1931246	2.13709908	0.0560255181	3.13885868E-03
200 °C	2.1931246	2.13709908	0.0560255181	3.13885868E-03
200 °C	2.1931246	2.13709908	0.0560255181	3.13885868E-03
200 °C	2.30963017	2.13709908	0.172531087	0.029766976
200 °C	2.35793485	2.13709908	0.220835768	0.0487684363
Sum	—	—	—	0.438733372

Limits for other percent confidence levels can be easily calculated by inserting the corresponding t or t' values into the above equations. All other values in the equations remain the same; only the t value is changed.

Annex 3

(Informative)

Applicability of Arrhenius Model Test for Linearity

A3.1 F-Test

The *F*-Test (for the statistician Fisher) [3] is based on the premise that in a truly linear relationship the variance of the sample life times at each temperature about the empirical regression line (designated as s_L^2) should be the same as the pooled variance of the sample life times at each temperature (designated as s_i^2).

The ratio, $F = s_L^2/s_i^2$, is compared to the tabulated values of F_{crit} in Table A3-1 at the appropriate degrees of freedom and if $F > F_{crit}$, then the null hypothesis of linearity is rejected.

Specifically, the variance of the *Y* values at each temperature, *i*, is calculated with Eq A3-1, or obtained by squaring each side of Eq 4C (section 2.2):

$$s_i^2 = \frac{n_i \Sigma Y_i^2 - (\Sigma Y_i)^2}{n_i(n_i - 1)} \qquad \text{(A3-1)}$$

The sums are to be taken over all the Y_i values at each test temperature, and the weighted mean of these variances calculated:

$$s_i^2 = \frac{\Sigma(n_i - 1)s_i^2}{\Sigma(n_i - 1)} \qquad \text{(A3-2)}$$

where n_i are the number of specimens at each temperature, *i*. The variance, s_L^2 of the Y_i values about corresponding points, m_i, on the regression line is calculated with Eq A3-3:

$$s_L^2 = \frac{\Sigma n_i(\overline{Y}_i - m_i)^2}{I - 2} = \frac{\Sigma n_i(\overline{Y}_i - (a + bX_i))^2}{I - 2} \qquad \text{(A3-3)}$$

where *a* and *b* are the intercept and slope of the regression line and *I* is the number of test temperatures.

Table A3-1 —Critical Values of F at 5% Significance Level

	k-2								
E(n_i-1)	1	2	3	4	5	6	8	12	24
1	161.40	199.50	215.70	224.60	230.20	234.00	238.90	243.90	249.00
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.61
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50
13	4.67	3.80	3.41	3.18	3.02	2.92	2.77	2.60	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35
15	4.54	3.68	329	3.06	2.90	2.79	2.64	2.48	2.29
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24
17	4.45	3.59	320	2.96	2.81	2.70	2.55	2.38	2.19
18	4.41	3.55	3.15	2.93	2.77	2.66	2.51	2.34	2.15
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.11
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	22.5	2.05
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.03
23	4.28	3.42	3.03	2.80	2.64	2.53	2.38	2.20	2.00
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	1.98
25	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79
60	4.00	3.18	2.76	2.53	2.37	2.25	2.10	1.92	1.70
120	3.92	3.07	2.68	2.45	229	2.18	2.02	1.83	1.61
∞	3.84	3.00	2.60	2.37	2.21	2.10	1.94	1.75	1.52

The ratio of $s_L{}^2$ to $s_I{}^2$ is the F value:

$$F = s_L^2 / s_1^2 \tag{A3-4}$$

This should be compared to the values in Table A3-1 for critical values of F at the 5% significance level. Enter the table at the column for I-2 and the row for the degrees of freedom for all individual tests values: $\Sigma\ (n_{\mathrm{i}} - 1)$, which is the denominator in Eq A3-2.

If the calculated F value is less than the value in Table A3-1 at I-2 and $\Sigma(n_{\mathrm{i}}- 1)$, there is less than 0.05 probability that the data do not correctly fit a linear (ie, Arrhenius) relation. The calculated F value and the corresponding F value from the table should be reported together with a statement of significance.

Although the F-Test procedure is reasonably simple and statistically valid, it may reject data that were acquired at considerable expense and time, and from which useful information can nevertheless be gleaned. This is particularly the case for tightly clustered data at each temperature (s_1 is small), suggesting good experimental results but which yield higher F values than would more widely scattered—and presumably poorer—experimental data.

In recognition of this, at least one forthcoming revision of IEC Publication 216-3 (see Annex 1, A1.2) permits a data "rescue" operation that is effected by an artificial increase of s_1. The limits to this increase are determined by further constraints, primarily by the maximum permissible difference between the empirical regression line and its lower 95% confidence bound.